

# TOWARDS HYPERGRAVITY

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The aim of this paper is to discuss a kinematical algebraic structure of a theory of gravity, that would be unitary, renormalizable and coupled in the same manner to both spinorial and tensorial matter fields. An analysis of the common features as well as differences of the Yang-Mills theories and gauge theories of gravity is carried out. In particular, we consider the following issues: (i) Representations of the relevant global symmetry on states and on fields, (ii) Relations between the relevant global and local symmetries, (iii) Representations of the local symmetries on states and fields, (iv) Dimensional analysis of the gauge algebra generators and the number of counter terms, and (v) Coupling to the spinorial matter fields. We conclude, that various difficulties on the gravity side can be overcome by considering the below outlined Hypergravity gauge theory, that to some extent parallels the string/membrane theories. This theory is based on an infinite Lie algebraic structure of generators that transforms as "states" of the infinite-dimensional irreducible representation of the  $\overline{SL}(4, R)$  subgroup of the Group of General Coordinate Transformations of  $R^4$ . The metric-affine and Poincaré gauge theories of gravity are obtained through a spontaneous symmetry breaking mechanism, with the metric field as nonlinear symmetry realizer.

The task of formulating a unitary renormalizable theory of Quantum Gravity is apparently the most outstanding one in contemporary Theoretical Physics. Numerous various field-theoretic approaches have been proposed, however each one fails to meet all basic physical and/or mathematical requirements. Simultaneous feature of unitary and renormalizability of such theories has been one of the most difficult requirements. We discuss, in the following, certain number of relevant algebraic questions in the Yang-Mills and Gauge Theories of Gravity, and suggest a Theory of Gravity that could evade common algebraic obstacles of the known Theories of Gravity.

## 1 Global Symmetry Representations on States and on Fields

There is a significant difference between the representations of the internal and space-time symmetries already at the level of global symmetries. Representations of an internal symmetry on states and on fields are mutually equivalent. They are of the same dimensionality, and both are unitary representations. In the case of global space-time symmetries, the representations on states are essentially (besides straightforward unitary infinite-dimensional 3-translation representations) given by the relevant little group unitary representations. In the Poincaré case the relevant little groups are either  $\overline{SO}(1, 3)$  or  $\overline{E}(2) \supset U(1)$ , yielding  $2J+1$  or 1-dimensional unitary representations respectively. In the case of Affine symmetry, the relevant little groups are  $\overline{SL}(3, R)$  or  $T_3 \wedge \overline{SL}(2, R)$ . The corresponding unitary representations are infinite-dimensional. The representations on fields are given by the representations of the non Abelian subgroup, i.e. by  $\overline{SO}(1, 3)$  in the Poincaré case, and  $\overline{SL}(4, R)$  in

the Affine case. The former ones are finite dimensional and non-unitary, while the latter ones are infinite-dimensional and unitary.

## 2 Global vs. Local Symmetries

The gauged version of a global internal symmetry group is given by a continuous direct product of groups that have identical Lie algebra structure as the starting global symmetry. In other words  $[X_a, X_b] = if_{ab}^c X_c \rightarrow [X_a(x), X_b(x)] = if_{ab}^c X_c(x)$ ,  $a, b, c = 1, 2, \dots, n$ , with the same structure constants. The gauging of a space-time symmetry yields a local space-time symmetry that, as a rule, differs considerably from the starting global symmetry. Let us consider the Affine symmetry generated by the momentum generators  $P_a$  and the shear generators  $Q_{ab}$ ,  $a, b = 0, 1, 2, 3$ . The Poincaré symmetry subgroup is generated by  $P_a$  and  $M_{ab} = Q_{[ab]}$ . In the process of gauging one introduces the parallel transport operators  $D_a$ , that replace the translations generators  $P_a$ . The commutation relations for  $[Q_{ab}(x), Q_{cd}(x)]$  and for  $[Q_{ab}(x), D_c(x)]$  are, up to  $x$  dependence, isomorphic to the corresponding global ones,  $[Q_{ab}, Q_{cd}]$  and  $[Q_{ab}, P_c]$  respectively. However, the commutator  $[P_a, P_b] = 0$  is replaced in the local case by<sup>1 2</sup>  $[D_a(x), D_b(x)] = e_a^\mu e_b^\nu (R_{\mu\nu}^{cd}(x) Q_{cd}(x) - T_{\mu\nu}^c(x) D_c(x))$ . Here, not only that the type of the commutation relations is changed, but instead of the structure constants one has structure functions - the affine curvature  $R^{ab}$  and torsion  $T^a$ . Thus, by gauging a finite-parameter space-time Lie group one obtains an infinite-parameter Lie group, as most easily seen by expanding  $R^{ab}$  and  $T^a$  in power series.

## 3 Local Symmetry Representations on States and on Fields

Owing to the fact that global and local internal symmetries are (up to  $x$  dependence) isomorphic, the corresponding representations on states and on fields are all mutually equivalent. The inhomogeneous transformation law in the local case relates to the space-time dependence only. The local space-time symmetry representations on states is achieved, by first selecting a form of the asymptotic space-time (e.g. Minkowskian), thus defining a certain subgroup (e.g. Poincaré), and then realizing non-linearly the infinite-parameter local symmetry group over the states of the starting global symmetry. As for the representations on fields, one makes use of the non-linear representations of the local symmetry group in the space of finite-dimensional world tensors. Representations on states and on fields are again a priori unrelated, and there are quite a number of unphysical field components due to the manifest covariance requirement.

## 4 Local Symmetry Generators Dimension

In contradistinction to the internal symmetry generators that are dimensionless, the generators of the local space-time symmetry group carry nontrivial dimension as easily seen from the Ogievetsky algebra generators expressions  $\{P_\mu, F_\mu^{\nu_1, \nu_2 \dots \nu_n} \sim x^{\nu_1} x^{\nu_2} \dots x^{\nu_n} P_\mu\}$ , i.e.  $[P_\mu] = l^{-1}$ ,  $[F_\mu^{\nu_1, \nu_2 \dots \nu_n}] = l^{n-1}$ . The  $F_\mu^\nu = e_\mu^a e^{b\nu} Q_{ab}$  are dimensionless only. The non-trivial dimensionality of the local space-time symmetry

generators is related to the appearance of dimensional coupling constants as well as of various types of counter terms in the quantum case. Moreover, the non-trivial field-dependent integration measure, due to non-linear realization of the full infinite symmetry, yields an infinite number of possible vertices, and thus an a priori non-renormalizable quantum theory.

## 5 Tensorial and Spinorial Matter Fields

In contradistinction to the case of internal symmetries, where all representations are a priori on the same footing, there has been quite a confusion even about the very existence of the spinorial representations of the local space-time symmetries<sup>3 4</sup>. The "world spinorial" representations<sup>5 6</sup> do exist and have been explicitly constructed in terms of the  $\overline{SL}(4, R)$  subgroup infinite-component spinorial representations.

## 6 Hypergravity

Consider, for simplicity, the  $\overline{SL}(3, R) \subset \overline{Diff}(3, R)$  group generated by the angular momentum  $J$  and shear  $T$ . The states of the corresponding infinite-dimensional representations<sup>7</sup> are labeled by  $(j; km)$ ,  $j = 1, 2, \dots, |k, m| \leq j$ . We introduce an infinite set of operators  $L_A = L_{(j \ k \ m)}$ , and require a Lie algebra closure among them. The resulting algebra is given by:

$$[L_{(j' \ k' \ m')}, L_{(j'' \ k'' \ m'')}] = \sum_{j \ k \ m} \left(1 - (-1)^{j'+j''-j}\right) \Delta(j \ j' \ j'') \begin{pmatrix} j & j' & j'' \\ -k & k' & k'' \end{pmatrix} \begin{pmatrix} j & j' & j'' \\ -m & m' & m'' \end{pmatrix} L_{(j \ k \ m)}$$

To each generator we assign a gauge potential  $\Gamma_\mu^A$  and define the corresponding Lie-algebra valued covariant derivative, as well as the corresponding field strengths  $R_{\mu\nu}^A$ . In this manner we obtain a Yang-Mills-like theory of gravity based on an infinite Lie algebra that realizes linearly the group of General Coordinate Transformations. As for the unitarity question we point out that the representations on states and on fields in this theory are directly related, while the renormalizability is boiled down to a finite number of counter terms - the infinite Lie algebra of the theory interrelates all possible vertices.

## References

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